Primes in the discriminant of curves of genus 3

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As you may know ...

$g = 0$

conics: $Q \simeq \mathbb{P}^1$

$y^2 = xz$

dim 0

$g = 1$

elliptic curves

$y^2 = x^3 + ax + b$

dim 1, j-invariants

$$H_D(x) = \prod_{E \text{ CM by } \mathcal{O}_D} (x - j(E)) \in \mathbb{Z}[x]$$

Genus 2 curves

They are still hyperelliptic curves $y^2 = f(x)$ with $\deg(f) = 5, 6$.

The moduli space has dimension 3: **Igusa invariants**.

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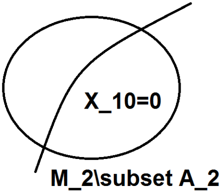

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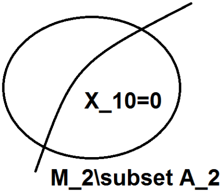

It makes sense to talk about the reduction of the invariants (Liu)

Curves of genus 3

$y^2 = f(x)$, def$(f) = 7, 8$, **dim 5**

Shioda invariants for
hyperelliptic curves

$F(x, y, z) = 0$ plane quartics

Dixmier-Ohno invariants: **dim** 6

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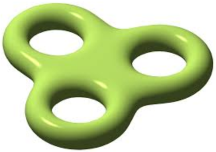
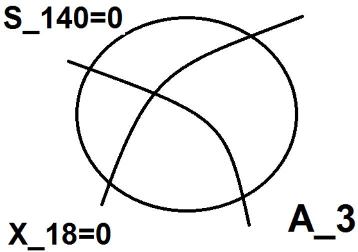


Hyperelliptic reduction: $\chi_{18} = 0$
Bad reduction: $\chi_{18} = \Sigma_{140} = 0$



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(Joint work with A. Fiorentino, R. Lercier, C. Ritzenthaler, J. Sijsling)

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- Extra primes: $\to$minimal discriminant

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For hyperelliptic curves: bad reduction has co-dimension 1.

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Primes in denominators $\iff$ primes of bad reduction. Bounds in:

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Exponents $\longrightarrow$ sol. to the embedding problem

Ionica, Kilicer, Lauter, L., Manzateanu, Massierer, Vincent; in progress

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Results à la Deuring: $E/\mathbb{F}_q$, $u/\mathbb{F}_q \to \tilde{E}/K$, $\tilde{u}/K$

Thank you!